Time : 5 Hour | STD 11 Maths | Total Marks : 250

kd 90+ ch-13 statistics

*** Choose the right answer from the given options. [1 Marks Each]** **[71]**

1. If 150 is the mean of 200 observations and 100 is the mean of some 300 other observations, find the mean of the combination:

   (A) 90 (B) 100 (C) 120 (D) 120

Ans. :

c. 120

Solution:

Mean of 200 observations = 150

Sum of 200 observations = 200 × 150 = 30000

Mean of 300 observations = 100

Sum of 300 observations = 300 × 100 = 30000

Total Sum = 30000 + 30000 = 60000

Number of observations = 100 + 200 = 500

$\text{Mean} = \frac{\text{Sum}}{\text{Number of observations}}$

$= \frac{60000}{500} = 120$

2. The following data has been arranged in ascending order. If their median is 63, find the value of x.34, 37, 53, 55, x, x + 2, 77, 83, 89 and 100.

   (A) 65 (B) 68 (C) 62 (D) 62

Ans. :

c. 62

Solution:

The series in ascending order is: 34, 37, 53, 55, x, x + 2, 77, 83, 89 and

The series has 10 numbers, even numbers.

Hence, the median will be the mean of the two middle numbers:

median = mean of 5^{th} and 6^{th} terms

$63 = \frac{x + x + 2}{2}$

$1260 = 2x + 2$

$2x = 124$

$x = 62$

3. The mean of 100 observations is 50 and their standard deviation is 5. The sum of all squares of all the observations is:

   (A) 50,000 (B) 250,000 (C) 252500 (D) 252500

Ans. :

c. 252500

Solution:

Let $\overline{x}$ and σ be the mean and standard deviation of 100 observations, respectively.

$\therefore \overline{x} = 50$, $\sigma = 5$ and n = 100

$\text{Mean}, \ \overline{x} = 50$

$\Rightarrow \frac{\sum x_i}{100} = 50$

$\Rightarrow \sum x_i = 5000 \ldots (1)$

Now,

Standard deviation, $\sigma = 5$

$\Rightarrow \sqrt{\frac{\sum x_i^2}{100} - \left(\frac{\sum x_i}{100}\right)^2} = 5$

$\Rightarrow \frac{\sum x_i^2}{100} - \left(\frac{5000}{100}\right)^2 = 25$ [From (1)]

$\Rightarrow \frac{\sum x_i^2}{100} = 25 + 2500 = 2525$

$\Rightarrow \sum x_i^2 = 252500$

Thus, the sum of all squares of all the observations is 252500.

Hence, the correct answer is option (c).

4. The sum $\sum_{r=1}^{10} \left(r^2 + 1\right) \times (r\,!)$ is equal to:

(A) (11)! (B) 10 × (11)! (C) 101 × (10)! (D) 101 × (10)!

Ans. :

b. 10 × (11)!

Solution:

$\sum (r^2 + 1) r! = \sum [r(r\ + 1) - (r - 1) r!$

$= \sum_{r=1}^{10} [r(r\ + 1)! - (r - 1) r!]$

$= (1 \times 2! - 0 \times 1!) + (2 \times 3! - 1 \times 2!) + \ldots\ldots + (10 \times 11! - 9 \times 10!)$

$= 10 \times 11!$

5. The daily sale of kerosene (in litres) in a ration shop for six days is as follows: 75, 120, 12, 50, 70.5 and 140.5 The average daily sale is:

(A) 150 (B) 10 (C) 142 (D) 142

Ans. :

d. 78

Solution:

$\text{Mean} = \frac{75+120+12+50+70.5+140.5}{6}$

= 78 The average daily sale is therefore the mean = 78.

6. The mean of the cubes of the first n natural numbers is:

(A) $\frac{n(n+1)^2}{2}$ (B) $\frac{n(n+1)^2}{4}$

(C) $\frac{n(n+1)(n+2)}{8}$ (D) $\frac{n(n+1)(n+2)}{8}$

Ans. :

b. $\frac{n(n+1)^2}{4}$

Solution:

First n natural numbers are 1,2,3,4,......,n

$\therefore \text{Mean} = \frac{1^3+2^3+3^3+\ldots\ldots+n^3}{n}$

Sum of the cubes of n natural numbers $= \left(\frac{n(n+1)}{2}\right)^2$

$\therefore \text{Mean} = \left(\frac{n(n+1)}{2}\right)^2 \times \frac{1}{n}$

$= \frac{n(n+1)^2}{4}$

7. Mean of 10 values is 32.6. If another values is included the mean becomes 31. The included value is:

(A) 16 (B) 14 (C) 15 (D) 15

Ans. :

c. 15

Solution:

Included value = 31 × 11 − 32.6 × 10 = 15

8. The mean of 864, 874, 884, 1000 and 1008 is:

(A) 928 (B) 1010 (C) 926 (D) 926

Ans. :

c. 926

Solution:

Formula,

$\frac{\sum x}{N} = \frac{864+874+884+1000+1008}{5}$

$= \frac{4630}{5}$

$= 926$

9. The attendance of a class of 45 boys for 10 days is given as 40, 30, 35, 45, 44, 41, 38, 44 and 41 then the mean attendance of a class is:

(A) 39 (B) 40 (C) 41 (D) 41

Ans. :

b. 40

Solution:

In this question one day attendance not givenGiven attendance as per Answer.

are 40, 42, 30, 35, 45, 44, 41, 38, 44 and 41 Then mean

$= \frac{40+42+30+35+45+44+41+38+44+41}{10}$

$= \frac{400}{10} = 40$

10. The following observations have been arranged in ascending order. If the median of the data is 78, find the value of x.

    44, 47, 63, 65, x + 13, 87, 93, 99, 110.

    (A) 65 (B) 68 (C) 66 (D) 66

Ans. :

a. 65

Solution:

The series in ascending order is: 44, 47, 63, 65, x + 13, 87, 93, 99, 110.

The series has 9 observations.

hence, the middle observation will be the median of the series.

Here, x + 13 is the middle observation

Therefore, x + 13 = 78

x = 65

11. Means of a set of 60 values is 23, if 4 is added to each these values the the new mean is:

    (A) 27 (B) 25 (C) 64 (D) 64

Ans. :

a. 27

Solution:

New mean = x~ = 23 + 4 = 27

12. The mean age of 30 student is 9 years. If the age of their teacher is included, it becomes 10 years. The age of teacher (in years) is:

    (A) 27 (B) 31 (C) 35 (D) 35

Ans. :

d. 40

Solution:

Given:Average age of 30 students = 9years.

Total age of 3030 students = 9 × 30 = 270 years. Teachers age included.

So, average age of 30 students + one teacher = 10years.

⇒ Total age of 30 students + one teacher = 10 × 31 = 310 years.

∴ age of teacher = 310 - 270 = 40 years.

13. The sum of the squares deviations for 10 observations taken from their mean 50 is 250. The coefficient of variation is:

(A) 10% (B) 40% (C) 50% (D) 50%

Ans. :

a. 10%

Solution:

We have:

$\overline{X} = 50,\ n = 10$

$\sum\limits_{i=1}^{10}\left(x_i - \overline{X}\right)^2 = 250$

$\therefore SD = \sqrt{\text{Variance of X}}$

$= \sqrt{\frac{\sum\limits_{i=1}^{10}\left(x_i - \overline{X}\right)^2}{n}}$

$= \sqrt{\frac{250}{10}}$

$= 5$

Using $CV = \frac{\sigma}{\overline{X}} \times 100$

$\Rightarrow CV = \frac{5}{50} \times 100$

$= 10\%$

14. In a class of 100 students there are 70 boys whose average marks in a subject are 75 If the average marks of whole class is 72 then what is the average marks of the girls?

(A) 73 (B) 65 (C) 68 (D) 68

Ans. :

a. 65

Solution:

Total students = 100 Average marks

=72 Total marks of the class = 72 × 100 = 7200

Total marks of the boys = 70 × 75 = 5250

Total marks of the girls = 7200 = 5250 = 1950

Average marks of the girls $= \frac{1950}{30} = 65$ hence, option B is correct.

15. Choose the correct answer.

The following information relates to a sample of size 60 $\sum x^2 = 18000$ and $\sum x = 960$, then the variance is:

(A) 6.63 (B) 16 (C) 22 (D) 22

Ans. :

d. 44

Solution:

We know that variance $(\sigma^2)\frac{\sum x_i^2}{N} - \left(\frac{\sum x_i}{N}\right)^2$

$= \frac{18000}{60} - \left(\frac{960}{60}\right)^2 = 300 - 256 = 44$

16. The mean deviation of the series a, a + d, a + 2d, ..., a + 2n from its mean is:

(A) $\frac{(n+1)d}{2n+1}$ (B) $\frac{nd}{2n+1}$ (C) $\frac{n(n+1)d}{2n+1}$ (D) $\frac{n(n+1)d}{2n+1}$

Ans. :

c. $\frac{n(n+1)d}{2n+1}$

Solution:

x_i	$\lvert x_i - \overline{X}\rvert = \lvert x_i - (a+nd)\rvert$
a	nd
a + d	(n - 1)d
a + 2d	(n - 2)d
a + 3d	(n - 3)d
:	:
:	:
a + (n + 1)d	d
a + nd	0
a + (n + 1)d	d
:	:
:	:
a + 2nd	nd
$\sum x_i = (2n+1)(a+nd)$	$\sum \lvert x_i - \overline{X}\rvert = n(n+1)d$

Therefore are 2n + 1 terms.

⇒ N = 2n + 1

$\sum x_i = a + a + d + a + 2d + a + 3d + \ldots + a + 2nd$

$= (2n+1)a + d(1+2+3+\ldots+2n)$ [a + a + a + ...(2n + 1)times = (2n + 1)a]

$= (2n+1)a + \frac{2n(2n+1)d}{2}$ $\Big[$Sum of the first n natural numbers is $\frac{n(n+1)}{2}$, but here we are considering$\Big]$

$= (2n+1)a + (2n+1)nd$

$= (2n+1)(a+nd)$

$\overline{X} = \frac{(2n+1)(a+nd)}{(2n+1)}$

$= a + nd$

$\sum \lvert x_i - \overline{X}\rvert = nd + (n-1)d + (n-2)d$

$+\ldots+d+0+d+2d+3d+\ldots+nd$

$= d(n+(n-1)+(n-2)+\ldots+1)$

$+0+d(1+2+3+\ldots.+n)$

$= \frac{dn(n+1)}{2} + \frac{dn(n+1)}{2} \quad \left\{ \because 1+2+3+\ldots.+n = \frac{n(n+1)}{2} \right\}$

$= n(n+1)d$

Mean deviation about the mean $= \frac{\sum \left| x_i - \overline{X} \right|}{N}$

$= \frac{n(n+1)d}{(2n+1)}$

17. Variance of the distribution 73, 77, 81, 85, ...,113 is:

(A) 10 (B) 160 (C) 161 (D) 161

Ans. :

c. 161

18. The average of monthly salary of fifteen employees in a company is Rs. 9450. If the supervisors salary is added, the average salary increase by Rs. 650 What is the salary of the supervisor?

(A) Rs.19,850 (B) Rs.20,050 (C) Rs. 20,250 (D) Rs. 20,250

Ans. :

b. Rs.20,050

Solution:

Average salary of 15 employees = Rs. 9450

Sum of the salaries of 15 employees = 15 × 9450 = 141750

New average after adding salary of supervisor = 9450 + 650 = 10100

Sum of salaries of 16 employees = 10100 × 16 = 1616600

Let the salary of the supervisor = x

Thus. x + 141750 = 161600

x = 19,850

19. Find the mean of:

9, 11, 12, 4 and 7

(A) 5.3 (B) 7.1 (C) 8.6 (D) 8.6

Ans. :

c. 8.6

Solution:

$\text{Mean} = \frac{9+11+12+4+7}{5}$

$\text{Mean} = \frac{43}{5} = 8.6$

20. Find the mean of the first five multiples of 7.

(A) 18 (B) 20 (C) 15 (D) 15

Ans. :

c. 15

Solution:

The first five multiples of 7 are 7, 14, 21, 28 and 35.

$$\text{Required mean } = \frac{7+14+21+28+35}{7} = \frac{105}{7} = 21$$

21. A measure of central location which splits the data set into two equal groups is called the:

(A) Mean (B) Mode (C) Median (D) Median

Ans. :

c. Median

Solution:

Median is the middle most value of a series. So it divides a series of observations into two equal parts where 50% of the observations are below. The median value and other 50% are above the median value.

22. A company produces on an average 4000 items per month for the first 3 months. How many items it must produce on an average per month over the next 9 months, to average 4375 items per month over the whole?

(A) 4500 (B) 4600 (C) 4680 (D) 4680

Ans. :

a. 4500

Solution:

Total production has to be 4375 × 12 = 52500

First three months production is 4000 × 3 = 12000

Total production has to be in remaining 9 months = 52500 - 12000 = 40500

Average production per month in remaining 9 months $= \frac{40500}{9} = 4500$

23. Kavita obtained 16, 14, 18 and 20 marks (out of 25) in maths in weekly test in the month of Jan 2000; then mean marks of Kavita is:

(A) 18 (B) 16.5 (C) 17 (D) 17

Ans. :

c. 17

Solution:

No. of test in the month Jan 2000 = 4 Total Marks obtained in 4 test

= 16 + 14 + 18 + 20

$68 \therefore \text{A.M} = \frac{\sum x}{n} = \frac{68}{4} = 17$

24. The mean of the squares of the first n natural numbers is:

(A) $n^2 + 1n2 + 1$

(B) $\frac{n^4+1}{n}$

(C) $\frac{(n+1)(2n+1)}{6}$

(D) $\frac{(n+1)(2n+1)}{6}$

Ans. :

c. $\frac{(n+1)(2n+1)}{6}$

Solution:

The first natural numbers are 1,2,3,......nTheir square are1$1^2, 2^2, 3^2 \ldots\ldots n^2$

$\text{Mean} = \frac{1^2 + 2^2 + 3^2 + \ldots\ldots\ldots + n^2}{n}$

$= \frac{n(n+1)(2n+1)}{6n}$

$\therefore$ Mean=n12 + 22 + 32 +........+ n2

$\therefore$ square of n natural numbers is$= \frac{n(n+1)(2n+1)}{6}$

$\text{Mean} = \frac{(n+1)(2n+1)}{6}$

25. The age of 13 school students are listed below. Find the median:
12, 9, 8, 13, 15, 14, 6, 18, 7, 11, 9, 14, 10

(A) 8 (B) 14 (C) 11 (D) 11

Ans. :

c. 11

Solution:

The median of a set of data is the middlemost number in the set.
So, first arrange the data in order.
6, 7, 8, 9, 10, 10, 11, 12, 13, 14, 14, 15, 18
The median is 11.

26. Given the list of numbers {1, 6, 3, 9, 16, 11, 2, 9, 5, 712, 13, 8} what is the median?

(A) 7 (B) 8 (C) 9 (D) 9

Ans. :

b. 8

Solution:

Given list is {1, 6, 3, 9, 16, 11, 2, 9, 5, 712, 13, 8} Arrange given set of integers in ascending order.
Then, we have 1, 2, 3, 5, 6, 8, 9, 11, 13, 16, 712
The middle number of the set is 8. Therefore the median is 8.

27. The mean of 8 numbers is 25 if each number is multiplied by 2 the new mean will be:

(A) 12.5 (B) 25 (C) 40 (D) 40

Ans. :

d. 50

Solution:

Mean of 8 numbers=25

$\therefore$ A.M $= \frac{\sum x}{n}$

$\Rightarrow 25 = \frac{\sum x}{8}$

$\Rightarrow \sum x = 25 \times 8 = 200$

If each number is multiply by 2 then new sum

$= 200 \times 2 = 400$

$\therefore \text{New mean} = 8400 = 50$

28. The difference between the maximum and the minimum obervations in data is called the ___________:

(A) Mean of the data
(B) Range of the data
(C) Mode of the data
(D) Mode of the data

Ans. :

b. Range of the data

Solution:

In arithmetic, the range of a set of data is the difference between the largest and smallest values.

So, difference between minimum and maximum values is called range.

29. If n = 10, $\overline{X} = 12$ and $\sum x_i^2 = 1530$, then the coefficient of variation is:

(A) 36%
(B) 41%
(C) 25%
(D) 25%

Ans. :

c. 25%

Solution:

Standard deviation is expressed in the following manner:

$\sigma = \sqrt{\frac{1}{n} \sum_i x_i^2 - (\overline{X})^2}$

$= \sqrt{\frac{1530}{10} - (12)^2}$

$= \sqrt{9}$

$= 3$

$\text{CV} = \frac{\sigma}{X} \times 100$

$= \frac{3}{12} \times 100$

$= 25$

30. The mean of 9 observations is 36. If the mean of the first 5 observations is 32 and that of the last 5 observations is 39, then the fifth observation is __________.

(A) 28
(B) 31
(C) 43
(D) 43

Ans. :

b. 31

Solution:

Mean of 9 observations = 36 ⇒ Sum of these 9 observations = 324

Sum of first five observations = 32 × 5 = 160

Sum of last five observations = 39 × 5 = 195

Fifth observation = Sum of first five observations + Sum of last five observations - Sum of all 9 observations

= 160 + 195 - 324 = 31

31. The standard deviation of first 10 natural numbers is:

(A) 5.5 (B) 3.87 (C) 2.97 (D) 2.97

Ans. :

d. 2.87

Solution:

We know that the standard deviation of first n natural number is $\sqrt{\frac{n^2-1}{12}}$.

$\therefore$ Standard deviation of first 10 natural numbers

$= \sqrt{\frac{10^2-1}{12}}$

$= \sqrt{\frac{99}{12}}$

$= \sqrt{8.25}$

$= 2.87$

Hence, the correct answer is option (d).

32. Choose the correct answer.

Consider the first 10 positive integers. If we multiply each number by -1 and then add 1 to each number, the variance of the numbers so obtained is:

(A) 8.25 (B) 6.5 (C) 3.87 (D) 3.87

Ans. :

a. 8.25

Solution:

Since, the first 10 positive integers are 1, 2, 3, 4, 5, 6, 7, 8, 9 and 10.

On multiplying each number by -1, we get -1, -2, -3, -4, -5, -6, -7, -8, -9, -10 On adding 1 in each number.

We get 0, -1, -2, -3, -4, -5, -6, -7, -8, -9.

$\therefore \sum x_i = -\frac{9\times10}{2} = -45$

and $\sum x_i^2 = 0^2 + (-1)^2 + (-2)^2 + \ \ldots\ + (9)^2 = \frac{9\times10\times19}{6} = 285$

$\mathrm{SD} = \sqrt{\frac{285}{10} - \left(\frac{-45}{10}\right)^2} = \sqrt{\frac{285}{10} - \frac{2025}{100}}$

$= \sqrt{\frac{2850-2025}{100}} = \sqrt{8.25}$

Now, $\text{variance} = (\mathrm{SD})^2 = \left(\sqrt{8.25}\right)^2 = 8.25$

33. If the mean of x + 2, 2x+ 3, 3x + 4, 4x + 5 is x + 2 then x is equal to:

(A) 0 (B) 1 (C) -1 (D) -1

Ans. :

c. -1

Solution:

Mean of the given distribution is,

$= \frac{(x+2)+(2x+3)+(3x+4)+(4x+5)}{4} = x+2$

= 4(x + 2) + (2x + 3) + (3x + 4) + (4x + 5) = x + 2, (given)

$= \frac{10x+14}{4} = x+2$

= 10x + 14 = 4x + 8 ⇒ x = -1

34. The average age of 6 students is 11 years. If two more students of age 14 and 16 years join, their average will become

(A) 13 years (B) 12 years (C) $12\frac{1}{2}$ years (D) $12\frac{1}{2}$ years

Ans. :

b. 12 years

Solution:

⇒ The average age of 66 students is 11 years.

⇒ Sum of age of 66 students = 6 × 11 = 66

⇒ When two more students of age 14 and 16 added to 6 students then, total students will become 8 ⇒ Sum of age of 8 students = 66 + 14 + 16 = 96

⇒ Required average $= \frac{96}{8} = 12$ years

35. Mode of the distribution is that value of the variate for which the_____is_____.

(A) frequency, maximum (B) Frequency, minimum

(C) frequency, arithmetic mean (D) frequency, arithmetic mean

Ans. :

a. frequency, maximum

Solution:

Mode of the distribution is that value of the variate for which the frequency is maximum.

36. The average of the first five odd prime numbers is:

(A) 7 (B) 7.8 (C) 8 (D) 8

Ans. :

b. 7.8

Solution:

Required average $= \frac{3+5+7+11+13}{5} = \frac{39}{5} = 7.8$

37. The wickets taken by a bowler in a one day cricket match are 4, 5, 6, 3, 4, 0, 3, 2, 3, 5. The mode of the data is ________ .

(A) 3 (B) 4 (C) 5 (D) 5

Ans. :

a. 3

Solution:

Mode of the set of data is the observation which occurs the most.

4, 6 occurs 2 times each, 6, 2 and 00 occurs 11 time each, whereas 3 occurs 3 times.

Thus, the number 33 occurs the maximum number of times i.e., 3. Therefore, mode of the data is 33.

38. Mean of twenty observations is 15. It two observations 3 and 14 are replaced by 8 and 9 respectively, then the new mean will be:

(A) 14 (B) 15 (C) 16 (D) 16

Ans. :

b. 15

Solution:

Total = 20 × 15 = 300

When observations 3 and 14 are replaced by 8 and 9 respectively,

new sum will be 300 - 3 - 14 + 8 + 9 = 300

New mean will be again 15

39. The average weight of 20 students was calculated 70kg. It was later discovered that one weight was misread as 70 instead of 90, the correct average in kg is

(A) 80 (B) 72 (C) 75 (D) 75

Ans. :

d. 71

Solution:

Wrong average =70

70 $\therefore$ Wrong sum of weight of 20 students = 20 × 70

$\therefore$ Correct sum of weights of 20 students

= 1400 - wrong weight + correct weight

= 1400 - 70 + 90 = 1420

$\therefore \text{Correct mean} = \frac{1420}{20} = 71\text{kg}.$

40. If for a sample of size 60, we have the following information $\sum x_i^2 = 18000$ and $\sum x_i = 960$ then the variance is:

(A) 6.63 (B) 16 (C) 22 (D) 22

Ans. :

d. 44

Solution:

Given $\sum x_i^2 = 18000,\ \sum x_i = 960$ and n = 60

$\therefore$ Variance

$= \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$= \frac{18000}{60} - \left(\frac{960}{60}\right)^2$

$= 300 - 256$

$= 44$

Hence, the correct answer is option (d).

41. The average age of two brothers is 9 years It is increased by 9 years when their mothers age is also included then the age of mother is:

(A) 35 years (B) 36 years (C) 37 years (D) 37 years

Ans. :

b. 36 years

Solution:

Average age of the two brother = 9 years

$\therefore$ Age of two brother = 9 × 2 = 18 years

If their mother age is included then the average age is increased by 9

$\therefore$ Average age of 3 = 9 + 9 = 18 years

Now Total age of three = 3 × 18 = 54 Years

$\therefore$ Mothers age = 54 - 18 = 36 years.

42. A grocer has a sale of Rs. 6435, Rs. 6927, Rs. 6855, Rs. 7230 and Rs. 6562Rs. for 5 consecutive months. How much sale must he have in the sixth month so that he gets an average sale of Rs. 6500?

(A) Rs. 4991 (B) Rs. 5991 (C) Rs. 6001 (D) Rs. 6001

Ans. :

a. Rs. 4991

Solution:

Total sale of 5 months = Rs. (6435 + 6927 + 7230 + 6562) = Rs. 34009.

Required sale = Rs. [(6500 × 6) - 34009]

= Rs. (39000 - 34009)

= Rs. 4991.

43. In a factory, the average salary of the employees is Rs. 70. If the average salary of 12 officers is Rs. 400 and that of the remaining employees is Rs. 60, then the number of employees are ....

(A) 396 (B) 400 (C) 408 (D) 408

Ans. :

c. 408

Solution:

⇒ Let total number of employees be

x ⇒ Average salary of total employee

= Rs. 70 = Average of 12 employees = Rs. 400 = Rs. 400 ⇒ Average of remaining employees

$\mathrm{Rs.60} \therefore 70 = \frac{400\times12+(\mathrm{x}-12)\times60}{\mathrm{x}}$

$\therefore 70\mathrm{x} = 4800 + 60\mathrm{x} - 720$

$\therefore 70\mathrm{x} = 4080 + 60\mathrm{x}$

$\therefore 10\mathrm{x} = 4080 \therefore \mathrm{x} = 408$

Total number of employees are 408.

44. The mean of 6 numbers is 42 If one number is excluded, the mean of remaining numbers is 45. Find the excluded number:

(A) 27 (B) 25 (C) 30 (D) 30

Ans. :

a. 27

Solution:

mean of 6 numbers = 42

Sum of 6 numbers = 42 × 6 = 252

After excluding one number,

mean of 5 numbers = 45

Sum of 5 numbers = 45 × 5 = 225

Thus, the number excluded = 252 - 225 = 27

45. The mean of 20 observations is 15 On checking it was found that the two observations were wrongly copied as 3 and 6. The correct values are 8 and 4 , then correct mean will be given by:

(A) 15.15 (B) 14.69 (C) 14.74 (D) 14.74

Ans. :

a. 15.15

Solution:

Mean of 20 observatios = 15

Sum of 20 observations = 15 × 20 = 300

Correct sum = 300 + 8 + 4 - 3 - 6 = 300

Correct mean $= \frac{303}{20} = 15.15$

46. Choose the correct answer.

Following are the marks obtained by 9 students in a mathematics test: 50, 69, 20, 33, 53, 39, 40, 65, 59 The mean deviation from the median is:

(A) 9 (B) 10.5 (C) 12.67 (D) 12.67

Ans. :

c. 12.67

Solution:

$\therefore$ Median $= 5^{\text{th}}$ term

$M_e = 50$

x_i	$d_i = \|x_i - M_e\|$
20	30
33	17
39	11
40	10
50	0
53	3
59	9
65	15
69	19
N = 2	$\sum d_i = 114$

$\therefore$ MD $= \frac{114}{9} = 12.67$

47. For a frequency distribution standard deviation is computed by applying the formula:

(A) $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$ (B) $\sigma = \sqrt{\left(\frac{\sum fd}{\sum f}\right)^2 - \frac{\sum fd^2}{\sum f}}$

(C) $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \frac{\sum fd}{\sum f}}$ (D) $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \frac{\sum fd}{\sum f}}$

Ans. :

a. $\sigma = \sqrt{\frac{\sum fd^2}{\sum f} - \left(\frac{\sum fd}{\sum f}\right)^2}$

48. A school has 20 teachers one of them retires at the age of 60 years and a new teacher replaces him this change reduces the average age of the staff by 2 years the age of new teacher is:

(A) 28 years (B) 25 years (C) 20 years (D) 20 years

Ans. :

c. 20 years

Solution:

Let the average age of the staff

= x Age of the new teacher

= y According to the questionNew age of the staff reduced by

$2\text{years} \Rightarrow \frac{20x - 60 + y}{20}$

$x - 2 \Rightarrow 20x - 60 + y$

$\Rightarrow 20x - 60 + y$

$= 20(x - 2) \Rightarrow 20x - 60 + y$

$\Rightarrow 20x - 60 + y$

$\Rightarrow y = 60 - 40 = 20$

$\Rightarrow y = 60 - 40$

= 20 Hence the age of the new teacher is 20 years.

49. Two high school classes took the same test. One class of 20 students made an average grade of 80%; the other class of 30 students made an average grade of 70%. The average grade for all students in both classes is:

(A) 75% (B) 74% (C) 77% (D) 77%

Ans. :

b. 74%

$\text{Average} = \frac{20.80+30.70}{20+30} = 74$

50. The captain of a cricket team of 11 members is 26 years old and the wicket keeper is 3 years older. If the ages of these two are excluded, the average age of the remaining players is one year less than the average age of the whole team. What is the average age of the team?

(A) 23 years (B) 24 years (C) 25 years (D) 25 years

Ans. :

a. 23 years

Solution:

Let the average age of the whole team by x years.

= 11 x - (26 + 29) = 9(x - 1)

= 11x - 9x = 46

= 2x = 46 ⇒ 2x = 46

= x = 23 ⇒ x = 23.

So, average age of the team is 23 years.

51. A child says that the median of 3, 14, 18, 20, 5 is 18. What concept does the child missed about finding the median?

(A) The order of numbers. (B) 14 (C) 18 (D) 18

Ans. :

a. The order of numbers.

Solution:

To calculate the median of any data series. The data series has to be arranged in the ascending order. The child hasn't arranged the data series in ascending order.

52. The most frequent value in a data set is?

(A) Median (B) Mode (C) Arithmetic mean (D) Arithmetic mean

Ans. :

b. Mode

Solution:

Mode is the highest occurring figure in a series.

It is the value in a series of observation that repeats maximum number of times and, which represents the whole series as most of the values, in the series revolves around this value.

Therefore, mode is the value that occurs the most frequent times in a series.

53. The average of four consecutive even numbers is one fourth of the sum of these numbers. What is the difference between the first and last number?

(A) 4 (B) 6 (C) 2 (D) 2

Ans. :

b. 6

Solution:

Let the numbers be 2x - 2, 2x, 2x + 2 and 2x + 4, where x is a natural number.

Then the difference between the first and last number = 2x + 4 - (2x - 2) = 6.

54. The average age of a teacher and three students is 20 years. If all students are of equal age and the difference between the age of the teacher and that of a student is 20 years, then the age of the teacher is:

(A) 25 years (B) 30 years (C) 35 years (D) 35 years

Ans. :

c. 35 years

Solution:

Let the age of each student be x years

Then, the age of teacher will be (x + 20) years

$\text{Mean age} = \frac{(x+20)+3x}{4}$

$20 = \frac{4x+20}{4}$

⇒ x = 15

Hence, age of the teacher = 35 years

55. Choose the correct answer.

Let x_1, x_2, x_3, x_4, x_5 be the observations with mean m and standard deviation s. The standard deviation of the observations kx_1, kx_2, kx_3, kx_4, kx_5 is:

(A) k + s (B) $\frac{s}{k}$ (C) ks (D) ks

Ans. :

c. ks

Solution:

Here, $m = \frac{\sum x_i}{N}$,

$$S = \sqrt{\frac{\sum x_i^2}{5} - \left(\frac{\sum x_i}{5}\right)^2}$$

$$\therefore\ SD = \sqrt{\frac{K^2 \sum x_i^2}{5} - \left(\frac{K \sum x_i}{5}\right)^2}$$

$$= \sqrt{\frac{K^2 \sum x_i^2}{5} - K^2\left(\frac{\sum x_i}{5}\right)^2}$$

$$= K = \sqrt{\frac{\sum x_i^2}{5} - \left(\frac{\sum x_i}{5}\right)^2}$$

$$= K.S$$

56. On Thursday, 20 of the 25 students in a chemistry class took a test and their average (arithmetic mean) was 80. On Friday, the other 5 students took the test and their average (arithmetic mean) was 90. What was the average for the entire class?

(A) 82 (B) 83 (C) 84 (D) 84

Ans. :

a. 82

Solution:

Average $= \frac{20(80)+5(90)}{25}$

$= \frac{1600+450}{25}$

$= \frac{2050}{25} = 82$

57. The standard deviation of the data:

x	1	a	a^2	....	a^n
f	${}^nC_0$	${}^nC_1$	${}^nC_2$	....	${}^nC_2$

is,

(A) $\left(\frac{1+a^2}{2}\right)^n - \left(\frac{1+a}{2}\right)^n$ (B) $\left(\frac{1+a^2}{2}\right)^{2n} - \left(\frac{1+a}{2}\right)^n$

(C) $\left(\frac{1+a^2}{2}\right)^{2n} - \left(\frac{1+a^2}{2}\right)^n$ (D) $\left(\frac{1+a^2}{2}\right)^{2n} - \left(\frac{1+a^2}{2}\right)^n$

Ans. :

d. None of these

Solution:

x_i	f_i	f_ix_i	x_i^2	$f_ix_i^2$
1	${}^nC_0$	${}^nC_0$	1	1
a	${}^nC_1$	$a\,{}^nC_1$	a^2	$a^2\,{}^nC_1$
a	${}^nC_2$	$a^2\,{}^nC_2$	a^4	$a^4\,{}^nC_2$
a	${}^nC_3$	$a^3\,{}^nC_3$	a^6	$a^6\,{}^nC_3$

:	:	:	:	:
a^n	${}^nC_n$	$a^n\,{}^nC_n$	a^{2n}	$a^{2n}\,{}^nC_n$
	$\sum_{i=1}^{n} f_i = 2^n$	$\sum_{i=1}^{n} f_i x_i = (1+a)^n$		$\sum_{i=1}^{n} f_i x_i^2 = (1+a^2)^n$

Number of terms, $N = \sum_{i=1}^{n} f_i = 2^n$

$$\sum_{i=1}^{n} f_i x_i = {}^n C_0 + a\,{}^nC_1 + a^2\,{}^nC_2 + \ldots + a^n\,{}^nC_n = (1+a)^n$$

$$\overline{X} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$$
$$= \frac{(1+a)^n}{2^n}$$
$$\sum_{i=1}^{n} f_i x_i^2 = (1+a^2)^n$$

$$\sigma^2 = \text{Variance}(X) = \frac{1}{N}\sum_{i=1}^{n} f_i x_i^2 - \left(\frac{\sum_{i=1}^{n} f_i x_i}{N}\right)^2$$

$$= \frac{(1+a^2)^n}{2^n} - \left[\frac{(1+a^2)^n}{2^n}\right]^2$$
$$= \left[\frac{1+a^2}{2}\right]^n - \left[\frac{1+a}{2}\right]^2 n$$
$$\sigma = \sqrt{\text{Variance}(X)}$$
$$= \sqrt{\left[\frac{1+a^2}{2}\right]^n - \left[\frac{1+a}{2}\right]^2 n}$$

58. The mean of 5 numbers is 18. If one number is excluded, their mean becomes 16. Then the excluded number is

(A) 18 (B) 25 (C) 26 (D) 30

Ans. : c

(c) Sum of total number = $18 \times 5 = 90$

After one number excluded

Sum of total number = $16 \times 4 = 64$

Then, excluded number is $90 - 64 = 26$.

59. Mean of 100 items is 49. It was discovered that three items which should have been $60, 70, 80$ were wrongly read as $40, 20, 50$ respectively. The correct mean is

(A) 48 (B) $82\frac{1}{2}$ (C) 50 (D) 80

Ans. : c

(c) Sum of 100 items $= 49 \times 100 = 4900$

Sum of items added$= 60 + 70 + 80 = 210$

Sum of items replaced$=40+20+50=110$

New sum $=4900+210-110=5000$

Correct mean$=\frac{5000}{100}=50$.

60. The $S.D.$ of 5 scores $1,2,3,4,5$ is

(A) $\frac{2}{5}$ (B) $\frac{3}{5}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

Ans. : c

(c) Mean $\bar{x}=\frac{1+2+3+4+5}{5}=3$

$S.D.$ = σ = $\sqrt{\frac{1}{n}\sum x_i^2-(\bar{x})^2}$

=$\sqrt{\frac{1}{5}(1+4+9+16+25)-9}$ =$\sqrt{11-9}=\sqrt{2}$.

61. If mean deviations about median of x , $2x$, $3x$, $4x$, $5x$, $6x$, $7x$, $8x$, $9x$, $10x$ is 30 , then $|x|$ equals

(A) 12 (B) 11 (C) 10 (D) 9

Ans. : a

Median is $(5.5\mathrm{x})=\mathrm{a}$

Mean deviation $=\frac{\sum|\mathrm{x_i}-\mathrm{a}|}{10}=30$

$\frac{2(4.5x+3.5x+2.5x+1.5x+.5x)}{10}=30$

$\frac{2(12.5)|\mathrm{x}|}{10}=30$

$|x|=12$

62. If the mean of the numbers $27+x$, $31+x$, $89+x$, $107+x$, $156+x$ is 82, then the mean of $130+x$, $126+x$, $68+x$, $50+x$, $1+x$ is

(A) 75 (B) 157 (C) 82 (D) 80

Ans. : a

(a) Given,

$82=\frac{(27+x)+(31+x)+(89+x)+(107+x)+(156+x)}{5}$

==> $82\times5=410+5x$ ==> $410-410=5x$

==> $x=0$

Required mean is,

$\bar{x}=\frac{130+x+126+x+68+x+50+x+1+x}{5}$

$\bar{x}=\frac{375+5x}{5}$

$=\frac{375+0}{5}=\frac{375}{5}$= 75.

63. The number of observations in a group is 40. If the average of first 10 is 4.5 and that of the remaining 30 is 3.5, then the average of the whole group is

(A) $\frac{1}{5}$ (B) $\frac{15}{4}$ (C) 4 (D) 8

Ans. : b

(b) $\frac{x_1+x_2+.....+x_{10}}{10}=4.5$

==> $x_1+x_2+.....+x_{10}=45$

$\frac{x_{11}+x_{12}+.....+x_{40}}{30}=3.5$

==> $x_{11}+x_{12}+.....+x_{40}=105$

$x_1+x_2+.....+x_{40}=150$

$\frac{x_1+x_2+.....+x_{40}}{40}$

$=\frac{150}{40}=\frac{15}{4}.$

64. In a series of $3n$ observations, if n observations are equal a and remaining observations are equal $-2a$, then the mean deviation of observations about their mean will be:-

(A) 0 (B) $\frac{a}{3}$ (C) $\frac{4a}{3}$ (D) $4a$

Ans. : c

Here, given observatios are a, a ...... n times, $-2a, -2a \ldots 2n$ times

No. of observations $=3\mathrm{n}$

mean $(\bar{X})=\frac{n\times a+2n\times(-2a)}{3n}=-a$

$\therefore$ Mean deviation about mean $=\frac{\Sigma|\mathrm{x}_1-\bar{\mathrm{x}}|}{3\mathrm{n}}$

$\frac{\mathrm{n}\times 2\mathrm{a}+2\mathrm{n}\times\mathrm{a}}{3\mathrm{n}}=\frac{4\mathrm{a}}{3}$

65. The mean deviation of the numbers $3,4,5,6,7$ is

(A) 0 (B) 1.2 (C) 5 (D) 25

Ans. : b

(b) $A.M.=\frac{3+4+5+6+7}{5}=5$

Mean deviation$=\frac{\Sigma|x_i-\bar{x}|}{n}$

$=\frac{|3-5|+|4-5|+|5-5|+|6-5|+|7-5|}{5}$

$=\frac{2+1+0+1+2}{5}=\frac{6}{5}=1.2.$

66. In a given frequency distribution, the respective values of mean and median are 21 and 22. The value of mode is

(A) 21.5 (B) 22 (C) 23.5 (D) 24

Ans. : d

Mode $=3$ median -2 mean

$=66-42=24$

67. If the algebraic sum of deviations of 20 observations from 30 is 20, then the mean of observations is

(A) 30 (B) 30.1 (C) 29 (D) 31

Ans. : d

(d) $\sum_{i=1}^{20}(x_i-30)=20$

==> $\sum_{i=1}^{20} x_i - 20 \times 30 = 20$

==> $\sum_{i=1}^{20} x_i = 620$.

Mean = $\frac{\sum_{i=1}^{20} x_i}{20}$

$= \frac{620}{20} = 31$.

68. The mean of 10 terms is 3 . If the first term is increased by 1 , second by 2 and so on, then the new mean is

(A) 4 (B) $\frac{17}{2}$ (C) 8 (D) $\frac{11}{2}$

Ans. : b

New mean

$= \frac{x_1+1+x_2+2+x_3+3+x_4+4+\ldots+x_{10}+10}{10}$

$= \frac{\sum x_i + \frac{10\times11}{2}}{10} = 3 + \frac{11}{2} = \frac{17}{2}$

69. The following data gives the distribution of height of studentsThe median of the distribution is

Height (in **cm**)	160	150	152	161	156	154	155
No of students	12	8	4	4	3	3	7

(A) 154 (B) 155 (C) 160 (D) 161

Ans. : b

(b)Arranging the data in ascending order of magnitude, we obtain

Height (in **cm**)	150	152	154	155	156	160	161
Number of students	8	4	3	7	3	12	4
Cumulative frequency	8	12	15	22	25	37	41

Here, total number of items is 41

$i.e.$, an odd number.

Hence, the median is $\frac{41+1}{2}^{th}$

$i.e.$, 21^{st} item.

From cumulative frequency table, we find that median

$i.e.$, 21^{st} item is 155,

(All items from 16 to 22^{nd} are equal, each 155).

70. Mean of 100 observations is 45. It was later found that two observations 19 and 31 were incorrectly recorded as 91 and 13. The correct mean is...

(A) 44 (B) 44.46 (C) 45 (D) 45.54

Ans. : b

(b) Sum of 100 items = $45 \times 100 = 4500$

Sum of items added = $19 + 31 = 50$

Sum of items replaced = $91 + 13 = 104$

New sum = $4500 - 104 + 50$ = 4446

New mean= $\frac{4446}{100}$ = 44.46

71. The mean weight per student in a group of seven students is $55\,kg$ If the individual weights of 6 students are $52, 58, 55, 53, 56$ and 54; then weights of the seventh student is.....kg

(A) 55 (B) 60 (C) 57 (D) 50

Ans. : c

(c) Total weight of 7 students is = $55 \times 7 = 385\ kg$

Sum of weight of 6 students

$= 52 + 58 + 55 + 53 + 56 + 54 = 328\ kg$

$\therefore$ Weight of seventh student = $385 - 328 = 57\ kg$.

*** Given section consists of questions of 2 marks each.** **[4]**

72. The mean and standard deviation of 20 observation is found to be 10 and 2 respectively. On rechecking, it was found that observation 8 was incorrect. Calculate the correct mean and standard deviation in cases of the wrong items is omitted.

Ans. : Here n = 20, $\bar{x} = 10$ and $\sigma = 2$

$\therefore\ \bar{x} = \frac{1}{n}\Sigma x_i \Rightarrow n \times \bar{x} = \Sigma x_i$

$\Rightarrow \Sigma x_i = 20 \times 10 = 200$

Therefore Incorrect $\Sigma x_i = 200$

Now $\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2 = \sigma^2$

$\Rightarrow \frac{1}{20}\Sigma x_i^2 - (10)^2 = 4 \Rightarrow \Sigma x_i^2 = 2080$

If wrong item is omitted.

When wrong item 8 is omitted from the data then we have 19 observations.

Therefore Correct Σx_i = Incorrect $\Sigma x_i - 8$

Correct $\Sigma x_i = 200 - 8 = 192$

Therefore Correct mean = $\frac{192}{19}$ = 10.1

Also correct Σx_i^2 = Incorrect $\Sigma x_i^2 - (8)^2$

$\Rightarrow$ Correct $\Sigma x_i^2 = 2080 - 64$ = 2016

Hence Correct variance $= \frac{1}{19}\left(correct\ \Sigma x_i^2\right)$ - (correct mean)^2

$= \frac{1}{19} \times 2016 - \left(\frac{192}{19}\right)^2$

$= \frac{2016}{19} - \frac{36864}{361} = \frac{38304-36884}{361} = \frac{1440}{361}$

Correct S.D. $= \sqrt{\frac{1440}{361}} = \sqrt{3.99}$ = 1.997

73. The mean and standard deviation of 20 observation are found to be 10 and 2 respectively. On rechecking, it was found that an observation 8 was incorrect. Calculate the correct mean and standard deviation in cases of it is replaced by 12 .

Ans. : Here we are given that, n = 20, $\bar{x} = 10$ and $\sigma = 2$

$\therefore\ \bar{x} = \frac{1}{n}\Sigma x_i \Rightarrow n \times \bar{x} = \Sigma x_i$

$\Rightarrow \Sigma x_i = 20 \times 10 = 200$

Therefore Incorrect $\Sigma x_i = 200$

Now $\frac{1}{n}\Sigma x_i^2 - (\bar{x})^2 = \sigma^2$

$\Rightarrow \frac{1}{20}\Sigma x_i^2 - (10)^2 = 4 \Rightarrow \Sigma x_i^2 = 2080$

If it is replaced by 12,

When wrong item 8 is replaced by 12

Therefore, Correct Σx_i = Incorrect Σx_i - 8 + 12

= 200 - 8 + 12 = 204

$\therefore$ Correct mean = $\frac{204}{20}$ = 10.2

Also correct Σx_i^2 = Incorrect Σx_i^2 - (8)^2 + (12)^2

= 2080 - 64 + 144 = 2160

$\therefore$ Correct variance $= \frac{1}{20}(correct\ \Sigma x_1^2)$ - (correct mean)^2

$= \frac{2160}{20} - \left(\frac{204}{20}\right)^2$

$= \frac{2160}{20} - \frac{41616}{400} = \frac{43200-41616}{400} = \frac{1584}{400}$

Correct S.D. $= \sqrt{\frac{1584}{400}} = \sqrt{3.96}$ = 1.989

*** Given section consists of questions of 3 marks each.** **[60]**

74. Find the mean deviation about the median for the data in: 13, 17, 16, 14, 11, 13, 10, 16, 11, 18, 12, 17.

Ans. : Arrange the data in ascending order, we have

10, 11, 11,12, 13, 13, 14, 16, 16, 17, 17, 18

Here n = 12 (which is even)

So median is average of 6 th and 7 th observations

$\therefore$ Median = $\frac{13+14}{2} = \frac{27}{2}$ = 13.5

x_i	$\lvert x_i - M \rvert$

10	3.5
11	2.5
11	2.5
12	1.5
13	0.5
13	0.5
14	0.5
16	2.5
16	2.5
17	3.5
17	3.5
18	4.5
Total	28

M.D. about median = $\frac{1}{n}\sum_{i=1}^{n}$ $|x_i - M|$

= $\frac{1}{12}\times$ 28 = 2.33

75. Find the mean deviation about the median for the data in: 36, 72, 46, 60, 45, 53, 46, 51, 49,42

Ans. : Arrange the data in ascending order, we have
36, 42, 45, 46, 49, 51, 53, 60, 72
Here n = 10 (which is even)

So median is average of 5 th and 6 th observation

$\therefore$ Median $= \frac{46+49}{2} = \frac{92}{2} = 47.5$

x_i	$\|x_i - m\|$
36	11.5
42	5.5
45	2.5
46	1.5
46	1.5
49	1.5
51	3.5
53	5.5

x_i	$\|x_i - m\|$
60	12.5
72	24.5
Total	70

M.D. about medican $= \frac{1}{n}\sum_{i-l}^{n}|x_i - M|$

$= \frac{1}{10} \times 70 = 7$

76. Find the mean deviation about the median for the data

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Ans. :

x_i	f_i	c.f.	$\|x_i - 7\|$	$f_i\|x_i - 7\|$
5	8	8	2	16
7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48
	26			84

$\frac{N}{2} = \frac{26}{2} = 13$

The c.f. just greater than 13 is 14 and corresponding value of x is 7.

∴ Median = 7

∴ M.D. about median $= \frac{1}{N}\sum f_i|x_i - M| = \frac{1}{26} \times 84 = 3.23$

77. Find the mean deviation from the median for the following data:

x_i	15	21	27	30	35
fi	3	5	6	7	8

Ans. :

x_i	f_i	Cum. Freq.	$\|d_i\| = \|x_i - 30\|$	$f_i\,\|d_i\|$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18

30	7	21	0	0
35	8	29	5	40
	29			Total = 148

$\frac{N}{2} = \frac{29}{2} = 14.5$

To calculate median we will locate the above value in column of cumulative frequency and the corresponding value of x_i will be our median.

Median = 30

$\mathrm{MD} = \frac{148}{29} = 5.10$

78. Find the mean deviation about the median for the following data.

x_i	3	6	9	12	13	15	21	22
f_i	3	4	5	2	4	5	4	3

Ans. :

79. Calculate the mean deviation about median for the following data.

Class	0-10	10-20	20-30	30-40	40-50	50-60
frequency	6	7	15	16	4	2

Ans. :

80. Find the variance of the following data:
6, 8, 10, 12, 14, 16, 18, 20, 22, 24

Ans. :

81. Find the variance and standard deviation of the following data.

x_i	4	8	11	17	20	24	32
f_i	3	5	9	5	4	3	1

Ans. :

82. Calculate the mean, variance and standard deviation for the following distribution:

Class	30-40	40-50	50-60	60-70	70-80	80-90	90-100
Frequency	3	7	12	15	8	3	2

Ans. :

83. Two plants A and B of a factory show following results about the number of workers and the wages paid to them:

	Plant A	**Plant B**
No. of workers	5000	6000

	Plant A	Plant B
Average monthly wages	₹ 2500	₹ 2500
Variance of distribution of wages	81	100

In which plant A or B is there greater variability in individual wages?

Ans. :

84. Coefficient of variation of the two distributions are 60 and 70 and their standard deviations are 21 and 16 respectively. What are their arithmetic means?

Ans. :

85. The following values are calculated in respect of heights and weights of the students of a section of Class XI:

	Height	Weight
Mean	162.6 cm	52.36 kg
Variance	127.69 cm^2	23.1361 kg^2

Find S.D and check which of them is more variable.

Ans. :

86. The variance of 20 observations is 5. If each observation is multiplied by 2, find the variance of the resulting observations.

Ans. :

87. The mean of 5 observations is 4.4 and their variance is 8.24. If three of the observations are 1, 2 and 6, find the other two observations.

Ans. :

88. If each of the observation $x_1, x_2, \ldots, x_n$ is increased by a, where a is a negative or positive number, then show that the variance remains unchanged.

Ans. :

89. For the frequency distribution:

x	2	3	4	5	6	7
f	4	9	16	14	11	6

Find the standard distribution.

Ans. :

x_i	f_i	f_ix_i	$f_ix_i^2$
2	4	8	16
3	9	27	81
4	16	64	256
5	14	70	350

6	11	66	396
7	6	42	294
	N = 60	$\sum f_i x_i = 277$	$\sum f_i x_i^2 = 1393$

$\therefore$ SD$\sqrt{\frac{\sum f_i x_i^2}{N} - \left(\frac{\sum f_i x_i}{N}\right)^2} = \sqrt{\frac{1393}{60} - \left(\frac{277}{60}\right)^2} = \sqrt{23.23 - (4.62)^2} = \sqrt{23.21 - 21.34}$

$= \sqrt{1.87} = 1.37$ Hence, the required SD = 1.37

90. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an even number.

Ans. : Consider first n natural number, when n is even i.e., 1, 2, 3, 4 ..... n.

$\therefore$ Mean $\bar{x} = \frac{1+2+3+ \text{......} +n}{n} = \frac{n(n+1)}{2n} = \frac{n+1}{2}$

$MD = \frac{1}{n}\left[\left|1 - \frac{n+1}{2}\right| + \left|2 - \frac{n+1}{2}\right| + \left|3 - \frac{n+1}{2}\right|\right] + \left|\frac{n-2}{2} - \frac{n+1}{2}\right|$

$+ \left|\frac{n}{2} - \frac{n+1}{2}\right| + \left|\frac{n+2}{2} - \frac{n+1}{2}\right| + \ldots\ldots + \left|n - \frac{n+1}{2}\right|$

$= \frac{1}{n}\left[\left|\frac{n-1}{2}\right| + \left|\frac{3n-n}{2}\right| + \left|\frac{5-n}{2}\right| + \ldots\ldots + \left|\frac{-3}{2}\right| + \left|\frac{1}{2}\right| + \ldots\ldots + \left|\frac{n-1}{2}\right|\right]$

$= \frac{2}{n}\left[\frac{1}{2} + \frac{3}{2} + \ldots\ldots + \frac{n-1}{2}\right]\left(\frac{n}{2}\right)$terms

$= \frac{1}{n} \cdot \left(\frac{n}{2}\right)^2$ [$\because$ Sum of first n natural numbers $= n^2$]

91. Two sets each of 20 observations, have the same standard derivation 5. The first set has a mean 17 and the second a mean 22. Determine the standard deviation of the set obtained by combining the given two sets.

Ans. : Given that $n_1 = 20, \sigma_1 = 5, \bar{x}_1 = 17$

and $n_2 = 20, \sigma_2 = 5, \bar{x}_2 = 22$

Now we know for combined two series that

$\sigma = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$

$= \sqrt{\frac{20 \times (5)^2 + 20 \times (5)^2}{20 + 20} + \frac{20 \times 20 (17 - 22)^2}{(20 + 20)^2}}$

$= \sqrt{\frac{1000}{40} + \frac{400 \times 25}{1600}}$

$= \sqrt{25 + \frac{25}{4}} = \sqrt{\frac{125}{4}}$

$= \sqrt{31.25} = 5.59$

Hence, the required SD = 5.59

92. If for a distribution $\sum(x-5) = 3, \sum(x-5)^2 = 43$ and the total number of item is 18, find the mean and standard deviation.

Ans. : Given, $n = 18,\ \sum(x-5) = 3,\ \sum(x-5)^2 = 43$

$\therefore$ Mean $= A + \frac{\sum(x-5)}{18} = 5 + \frac{3}{18}$

$= 5 + 0.1666 = 5.1666 = 5.17$

and SD $= \sqrt{\frac{\sum(x-5)^2}{n} - \left(\frac{\sum(x-5)}{n}\right)^2}$

$= \sqrt{\frac{43}{18} - \left(\frac{3}{18}\right)^2}$

$= \sqrt{2.3889 - (0.166)^2}$

$= \sqrt{2.3889 - 0.0277}$

$= 1.53$

93. Find the mean and variance of the frequency distribution given below:

x	$1 \leq x < 3$	$3 \leq x < 5$	$5 \leq x < 7$	$7 \leq x < 10$
f	6	4	5	1

Ans. :

x	f_i	X_i	$f_i x_i$	$f_i x_i^2$
1-3	6	2	12	24
3-5	4	4	16	64
5-7	5	6	30	180
7-10	1	8.5	8.5	72.25
Total	n = 16		$\sum f_i x_i = 66.5$	$\sum f_i x_i^2 = 340.25$

$\therefore$ Mean $= \frac{\sum f_i x_i}{\sum f_i} = \frac{66.5}{16} = 4.13$

And variance $= \sigma^2 = \frac{\sum f_i x_i^2}{\sum f_i} - \left(\frac{\sum f_i x_i}{\sum f_i}\right)^2$

$= \frac{340.25}{16} - (4.13)^2$

$= 21.2656 - 17.0569 = 4.21$

*** Given section consists of questions of 5 marks each.** **[115]**

94. The mean and variance of eight observations are 9 and 9.25 respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.

Ans. : Let two remaining observations be x and y. Then

$\frac{6+7+10+12+12+13+x+y}{8} = 9$

$\therefore$ 60 + x + y = 72 $\Rightarrow$ x + y = 12 ...(i)

Also $\frac{1}{8}(6^2 + 7^2 + 10^2 + 12^2 + 12^2 + 13^2 + x^2 + y^2) - (9)^2 = 9.25$

$\Rightarrow \frac{1}{8}(36 + 49 + 100 + 144 + 144 + 169 + x^2 + y^2) - 81 = 9.25$

$\Rightarrow 642 + x^2 + y^2 = 722$

$\Rightarrow x^2 + y^2 = 80$. . . (ii)

Now $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

$\Rightarrow (12)^2 + (x - y)^2 = 2 \times 80$

$\Rightarrow (x - y)^2 = 160 - 144 \Rightarrow (x - y)^2 = 16 \Rightarrow x - y = \pm 4$

When x - y = 4

Solving x + y = 12 and x - y = 4 we get x = 8 and y = 4

When x - y = - 4

Solving x + y = 12 and x - y = -4 we get x = 4 and y = 8

95. Find the mean, variance and standard deviation using short cut method.

Height in cm	70-75	75-80	80-85	85-90	90-95	95-100	100-105	105-110	110-115
No. of children	3	4	7	7	15	9	6	6	3

Ans. :

Height in cms.	Mid values x_i	f_i	$u=\frac{x-92.5}{5}$	fu	fu^2
70-75	72.5	3	- 4	- 12	48
75-80	77.5	4	- 3	- 12	36
80-85	82.5	7	- 2	- 14	28
85-90	87.5	7	- 1	- 7	7
90-95	92.5	15	0	0	0
95-100	97.5	9	1	9	9
100-105	102.5	6	2	12	24
105-110	107.5	6	3	18	54
110-115	112.5	3	4	12	48
		60		**6**	254

Mean $(\bar{x}) = A + \frac{\Sigma fu}{N} \times h = 92.5 + \frac{6}{60} \times 5$ = 92.5 + 0.5 = 93

Variance $(\sigma^2) = \frac{h^2}{N^2}[N\Sigma fu^2 - (\Sigma fu)^2]$

$= \frac{(5)^2}{(60)^2}[60 \times 254 - (6)^2]$

$= \frac{25}{3600}[15240 - 36] = \frac{25}{3600} \times 15204 = 105.58$

Standard deviation $(\sigma) = \sqrt{105.58} = 10.27$

96. The mean and variance of 7 observations are 8 and 16 respectively. If five of the observations are 2, 4, 10, 12, 14 find the remaining two observations.

Ans. : Let two remaining observations be x and y. Then

$\frac{2+4+10+12+14+x+y}{7} = 8$

$\therefore$ 42 + x + y = 56 $\Rightarrow$ x + y = 14

Also $\frac{1}{7}$ $(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2) - (8)^2 = 16$

$\Rightarrow \frac{1}{7}(4 + 16 + 100 + 144 + 196 + x^2 + y^2) - 64 = 16$

$\Rightarrow$ 460 + x^2 + y^2 = 560 $\Rightarrow$ x^2 + y^2 = 100 .... (ii)

Now $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

$\Rightarrow$ $(14)^2 + (x - y)^2 = 2 \times 100$

$\Rightarrow$ $(x - y)^2$ = 200 - 196 $\Rightarrow$ $(x - y)^2$ = 4 $\Rightarrow$ x - y = $\pm$2

When x - y = 2

Solving x + y = 14 and x - y = 2 we get x = 8 and y = 6

When x - y = -2

Solving x + y = 14 and x - y = - 2 we get x = 6 and y = 8

97. The mean and standard deviation of marks obtained by 50 students of a class in three subjects, Mathematics, Physics and Chemistry are given below:

Subject	Mathematics	Physics	Chemistry
Mean	42	32	40.9
Standard deviation	12	15	20

Which of these three subjects shows the highest variability in marks and which shows the lowest?

Ans. : Given, n = 50

For Mathematics

$\overline{x}$ = 42 and σ = 12

Coefficient of variation (CV)= $\frac{\sigma}{\overline{x}} \times$ 100 = $\frac{12}{42} \times 100$

= $\frac{2}{7} \times$ 100 = $\frac{200}{7}$ = 28.57 ...(i)

For Physics

$\overline{x}$ = 32 and σ = 15

Coefficient of variation (CV) = $\frac{\sigma}{\overline{x}} \times 100 = \frac{15}{32} \times 100$

= $\frac{1500}{32}$ = 46.87 ....(ii)

For Chemistry

$\overline{x}$ = 40.9 and σ = 20

Coefficient of variation (CV) = $\frac{\sigma}{\overline{x}} \times 100 = \frac{20}{40.9} \times 100$

= $\frac{2000}{40.9}$ = 48.89 ....(iii)

From equations (i), (ii) and (iii),

$\Rightarrow$ CV of Chemistry > CV of Physics > CV of Mathematics

$\therefore$ Chemistry shows the highest variability while Mathematics shows the least variability.

98. The mean and standard deviation of a group of 100 observation were found to be 20 and 3 respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations are omitted.

Ans. : Here n = 100, $\bar{x}$ = 20 and $\sigma = 3$

$\therefore\ \bar{x} = \frac{1}{n}\Sigma x_i \Rightarrow \Sigma x_i = n \times \bar{x} = 100 \times 20$ = 2000

$\therefore$ Incorrect Σx_i = 2000

Now $\frac{1}{n}\Sigma x_i^2 - (\bar{x}) = \sigma^2$

$\Rightarrow \frac{1}{100}\Sigma x_i^2 - (20)^2 = 9 \Rightarrow \Sigma x_i^2 = 40900$

When wrong items 21, 21 and 18 are omitted from the data , we have 97 observations.

Correct Σx_i = Incorrect Σx_i - 21 - 21 - 18

= 2000 - 21 - 21 - 18 = 1940

$\therefore$ Correct mean $= \frac{1940}{97}$= 20

Also correct Σx_i^2 = Incorrect Σx_i^2 - $(21)^2$ - $(21)^2$ - $(18)^2$

= 40900 - 441 - 441 - 324 = 39694

$\therefore$ Correct variance =$\frac{1}{97}$ (correct Σx_i^2) - (correct mean)^2

$= \frac{1}{97} \times 39694 - (20)^2$

= 409.22 - 400 = 9.22

Correct S.D. = $\sqrt{9.22}$ = 3.036

99. The mean and standard deviation of 100 observation were calculated as 40 and 5.1 respectively by a student who took by mistake 50 instead of 40 for one observation. What are the correct mean and standard deviation?

Ans. : We have,

$\mathrm{n} = 100, \overline{\mathrm{X}} = 40, \sigma = 5.1$

$\therefore \overline{\mathrm{X}} = \frac{1}{\mathrm{n}}\sum \mathrm{x_i} = \overline{\mathrm{X}} = 100 \times 40 = 4000.$

Corrected $\sum \mathrm{x_i}$ = Incorrected $\sum \mathrm{x_i}$ - (sum of incorrect values) + (sum of correct values)

= 4000 -50 + 40 = 3990

$\therefore \text{Corrected mean} = \frac{\text{corrected} \sum \mathrm{x_i}}{\mathrm{n}} = \frac{3990}{100} = 39.9$

Now $\sigma = 5.1$

$\Rightarrow 5.1^2 = \frac{1}{100}\left(\sum \mathrm{x_i}^2\right) - \left(\frac{1}{100}\sum \mathrm{x_i}\right)^2$

$\Rightarrow 26.01 = \frac{1}{100}\left(\sum \mathrm{x_i}^2\right) - \left(\frac{4000}{100}\right)^2$

$\Rightarrow 26.01 = \frac{1}{100}\left(\sum \mathrm{x_i}^2\right) - 1600$

$\sum \mathrm{x_i}^2 = 100 \times 1626.01 = 162601$

Incorrect $\sum \mathrm{x_i}^2 = 162601$

corrected $\sum \mathrm{x_i}^2$ = $\left(\text{incorrected} \sum \mathrm{x_i}^2\right)$ - (sum of squers of incorrect values) + (sum of squers of correct values)

= 162601 - $(50)^2$ + $(40)^2$ = 161701

so, Corrected $\sigma=\sqrt{\frac{1}{n}\sum x_i^2-\left(\frac{1}{n}\sum x_i\right)^2}=\sqrt{\frac{161701}{100}-\left(\frac{3990}{100}\right)^2}$

$=\sqrt{1617.01-1592.01}=5$

100. The mean and standard deviation of a group of 100 observations were found to be 20 and 3 respectively. Later on it was found that three observation were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observation were omitted.

Ans. : We have, $n=100, \bar{x}=20$ and $\sigma=3$

Since $\bar{x}=\frac{1}{n}\sum x_i$

$\Rightarrow \sum x_i=n\bar{x}=20\times 100=2000$

$\Rightarrow$ Incorrect $\sum x_i=2000$

and,

$\sigma=3$

$\Rightarrow \sigma^2=9$

$\Rightarrow \frac{1}{n}\sum x_i^2-(\text{Mean})^2=9$

$\Rightarrow \frac{1}{100}\sum x_i^2-400=9$

$\Rightarrow \sum x_i^2=409\times 100$

$\Rightarrow$ Incorrect $\sum x_i^2=40900.$

When the incorrect observation 21, 21, 18 are omitted from the data:

n = 97

Now, Incorrect $\sum x_i=2000$

$\Rightarrow \sum x_i=2000-21-21-18=1940$

and,

Incorrect $\sum x_i^2=40900$

$\Rightarrow$ Corrected $\sum x_i^2=40900-21^2-21^2-18^2$

$\Rightarrow$ Corrected $\sum x_i^2=40900-1206$

$\Rightarrow$ Corrected $\sum x_i^2=39694$

$\therefore$ Corrected mean $=\frac{1940}{97}=20$

$\Rightarrow$ Corrected variavce $=\frac{1}{97}\left(\text{Corrected}\sum x_i^2\right)-(\text{Corrected mean})^2$

$\Rightarrow$ Corrected variavce $=\frac{39694}{97}-(20)^2=409.22-400=9.22$

$\therefore$ Corrected standard deviation $=\sqrt{9.22}=3.04$

101. The mean and variance of 8 observation are 9 and 9.25 respectively. If six of the observation are 6, 7, 10, 12, 12 and 13, find the remaining two observation.

Ans. : Let x and y be the remaining two observations. Then,

Mean = 9

$\Rightarrow \frac{6+7+10+12+12+13+x+y}{8}=9$

⇒ 60 + x + y =72

⇒ x + y = 12.........(i)

Variance = 9.25

$\Rightarrow \frac{1}{8}\left(6^2+7^2+10^2+12^2+12^2+13^2+x^2+y^2\right)-(\text{Mean})^2=9.25$

$\Rightarrow \frac{1}{8}\left(36+49+100+144+144+169+x^2+y^2\right)-81=9.25$

⇒ 642 + x^2 + y^2 = 722

⇒ x^2 + y^2 = 80........(ii)

Now, $(x + y)^2 + (x - y)^2 = 2(x^2 + y^2)$

⇒ 144 + $(x - y)^2$ = 2 × 80

⇒ x - y = 16

$\Rightarrow x-y=\pm 4$

if x - y = 4, then x + y =12 and x - y = 4 ⇒ x = 8, y = 4

if x - y = -4, then x + y =12 and x - y = -4 ⇒ x = 4, y = 8

Hence, the remaining two observation are 4 and 8.

102. Find the numberof observation lying between $\overline{X}-\text{M.D.}$ and $\overline{X}+\text{M.D.}$ is the mean deviation from the mean.
22, 24, 30, 27, 29, 31, 25, 28, 41, 42

Ans. : Let $\overline{x}$ be the mean of the data set.

$\overline{x}=\frac{22+24+30+27+29+31+25+28+41+42}{10}=29.9$ =

x_i	$\lvert d_i \rvert = \lvert x_i - 29.9 \rvert$
22	7.9
24	5.9
30	0.1
27	2.9
29	0.9
31	1.1
25	4.9
28	1.9
41	11.9
42	12.1
Total	48.8

$\text{MD}=\frac{1}{10}\times 48.8=4.88$

$\overline{x}$ - M.D. = 29.9 - 4.88 = 25.02,

and, $\overline{x}$ + M.D. = 29.9 + 4.88 = 34.78

There are 5 observation between 25.02 and 34.78.

103. The mean and standard deviation of 6 observation are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observation.

Ans. : Mean = $\overline{X} = 8$

n = 6

$\sigma = \text{S.D} = 4$

If $x_1, x_2, \ldots\ldots x_6$ are the given observation

$$\overline{X} = \frac{1}{n} \times \sum_{i=1}^{6} x_i$$

$$\Rightarrow 8 = \frac{1}{6} \times \sum_{i=1}^{6} x_i$$

Let $u_1, u_2 \ldots\ldots u_6$ be the new observation

$\Rightarrow u_i = 3x_i$ (for i = 1, 2, 3…6)

$\Rightarrow$ Mean of new observation $= \overline{U} = \frac{1}{n} \times \sum_{i=1}^{6} u_i$

$$= \frac{1}{6} \times \sum_{i=1}^{6} 3x_i$$

$$= 3 \times \frac{1}{6} \times \sum_{i=1}^{6} x_i$$

$= 3\,\overline{X}$

$= 3 \times 8$

$= 24$

$\text{SD} = \sigma_x = 4$

${\sigma_x}^2$ = Variance X

$\therefore$ Variance X = 16

$$\Rightarrow \frac{1}{6} \sum_{i=1}^{6} (x_i - \overline{X})^2 = 16 \ldots\ldots(1)$$

$$\text{Variance (U)} = {\sigma_u}^2 = \frac{1}{6} \sum_{i=1}^{6} (u_i - \overline{U})^2$$

$$= \frac{1}{6} \times \sum_{i=1}^{6} (3x_i - 3\overline{X})^2$$

$$= 3^2 \times \frac{1}{6} \sum_{i=1}^{6} (x_i - \overline{X})^2$$

$= 9 \times 16$

$\sigma_u = \sqrt{\text{Variance (U)}}$

$= \sqrt{9 \times 16}$

$= 12$

104. While calculating the mean and variance of 10 readings, a student wrongly used the reading of 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance.

Ans. : Mean = 45

Variance = 16

n = 10

$\sum \mathrm{x_i} = 450$

Corrected Sum = 450 - 52 + 25 = 423

Corrected Mean = 42.3

Variance = 16

$16 = \frac{\sum \mathrm{x_i^2}}{10} - (45)^2$

Incorrected $\sum \mathrm{x_i^2} = 20410$

Corrected $\sum \mathrm{x_i^2} =$

Incorrect $\sum \mathrm{x_i^2}$ - (Sum of squares of incorrect value) + (Sum of squares of corrected value)

Corrected $\sum \mathrm{x_i^2} = 20410 - 2704 + 625 = 18331$

Corrected $\sigma = \sqrt{\frac{\text{Corrected} \sum \mathrm{x_i^2}}{\mathrm{n}} - (\text{Corrected Mean})^2}$

Corrected $\sigma = \sqrt{\frac{18331}{10} - (42.3)^2} = 6.62$

Corrected Variance = 6.62 × 6.62 = 43.82

105. Find the numberof observation lying between $\overline{\mathrm{X}} - \mathrm{M.D.}$ and $\overline{\mathrm{X}} - \mathrm{M.D.}$ is the mean deviation from the mean.

34, 66, 30, 38, 44, 50, 40, 60, 42, 51

Ans. : Let $\overline{\mathrm{x}}$ be the mean of the data set.

$\overline{\mathrm{x}} = \frac{34+66+30+38+44+50+40+60+42+51}{10} = 45.5$

$\mathrm{MD} = \frac{1}{\mathrm{n}} \sum_{\mathrm{i}=1}^{\mathrm{n}} |\mathrm{d_i}|, \mathrm{where} |\mathrm{d_i}| = |\mathrm{x_i} - \overline{\mathrm{x}}|$

$\mathbf{x_i}$	$\mathbf{\|d_i\| = \|x_i - 45.5\|}$
34	11.5
66	20.5
30	15.5
38	7.5
44	1.5
50	4.5
40	5.5
60	14.5
42	3.5
51	5.5
Total	90

$\mathrm{MD} = \frac{1}{10} \times 90 = 9$

$\overline{\mathrm{x}}$ - M.D. = 45.5 - 9 = 36.5

Also, $\overline{x}$ + M.D. = 45.5 + 9 = 54.5

Hence, there are 6 observations between 36.5 and 54.5.

106. Find the standard deviation for the following data:

x	2	3	4	5	6	7
f	4	9	16	14	11	6

Ans. :

x_i	f_i	$f_i x_i^2$
2	4	16
3	9	81
4	16	256
5	14	350
6	11	396
7	6	294
	N = 60	**Total = 1393**

Mean $= \frac{8+27+64+70+66+42}{60} = \frac{277}{60} = 4.62$

Var $= \frac{1393}{60} - (4.62)^2 = 1.88$

SD $= \sqrt{1.88} = 1.37$

107. The mean and standard deviation of 20 observation are found to be 10 and 2 respectively. On reacheking it was found that an observation 8 was incorrect. Calculate the correct and standard deviation in each of the following cases:

i. If wrong item is omitted.
ii. If it is replaced by 12.

Ans. : $\mathrm{n} = 20, \overline{\mathrm{X}} = 10, \sigma = 2$

$\therefore \overline{\mathrm{x}} = \frac{1}{\mathrm{n}} \sum \mathrm{x_i}$

$\Rightarrow \sum \mathrm{x_i} = \mathrm{n}\overline{\mathrm{x}} = 20 \times 10 = 200$

$\Rightarrow \text{Incorrected} \sum \mathrm{x_i} = 200$

and,

$\sigma = 2$

$\Rightarrow \sigma^2 = 4$

$\Rightarrow \frac{1}{\mathrm{n}} \sum \mathrm{x_i}^2 - (\text{Mean})^2 = 4$

$\Rightarrow \frac{1}{20} \sum \mathrm{x_i}^2 - 100 = 4$

$\Rightarrow \sum \mathrm{x_i}^2 = 104 \times 20$

$\Rightarrow \sum \mathrm{x_i}^2 = 2080.$

i. When 8 is omitted from the data:

If 8 is omitted from the data, then 19 observation are left.

Now, Incorrected $\sum \mathrm{x_i} = 200$

⇒ Corrected $\sum \mathrm{x_i}^2 + 8^2 = 2080$

⇒ Corrected $\sum {x_i}^2 = 2080 - 64$

⇒ Corrected $\sum x_i^2 = 2016$

$\therefore$ Corrected mean $= \frac{192}{19} = 10.10$

⇒ Corrected variance $= \frac{1}{19}\left(\text{corrected} \sum {x_i}^2\right) - \left(\text{Corrected mean}\right)^2$

⇒ Corrected variance $= \frac{2016}{19} - \left(\frac{192}{19}\right)^2$

Corrected variance $= \frac{38304-36864}{361} = \frac{1440}{361}$

$\therefore$ Corrected standard deviation $= \sqrt{\frac{1440}{361}} = \frac{12\sqrt{10}}{19} = 1.997$

ii. When the incorrect observation 8 is replaced by 12:

We have, Incorrected $\sum x_i = 200$

$\therefore$ Corrected $\sum x_i = 208 - 8 + 12 = 204$

and,

$\Rightarrow$ Incorrected $\sum {x_i}^2 = 2080$

$\therefore$ Corrected $\sum {x_i}^2 = 2080 - 8^2 + 12^2 = 2160$

Now, $\therefore$ Corrected mean $= \frac{204}{20} = 10.2$

Corrected variance $= \frac{1}{20}(\text{Corrected} \sum {x_i}^2) - (\text{Corrected mean})^2$

⇒ Corrected variance $= \frac{2160}{20} - \left(\frac{204}{20}\right)^2$

⇒ Corrected variance $= \frac{2160 \times 20 - (204)^2}{(20)^2}$

⇒ Corrected variance $= \frac{43200-41616}{400} = \frac{1584}{400}$

$\therefore$ Corrected standard deviation $= \sqrt{\frac{1584}{400}} = \frac{\sqrt{396}}{10} = 1.9899$

108. The variance of 20 observation is 5. If each observation is multiplied by 2, find the variance of the resulting observation.

Ans. : We have, n = 20, and $\sigma^2 = 5$

Now each observation is multiplied by 2.

Suposs X = 2x be the new data.

$\therefore \overline{X} = \frac{1}{20}\sum 2x_i = \frac{1}{20} \times 2 \sum x_i = 2\overline{x}$

$\Rightarrow \sum {X_i}^2 = 4 \sum {x_i}^2$

Since, $\sigma^2 = 5$

$\Rightarrow \frac{1}{n}\sum {x_i}^2 - (\overline{x})^2 = 5$

Now, for the new data

$\sigma^2 = \frac{1}{n}\sum {X_i}^2 - (\overline{x})^2 = 4\sum {x_i}^2 - (2\overline{x})^2 = 4\left(\sum {x_i}^2 - (\overline{x})^2\right) = 4 \times 5 = 20$

109. Calculate the mean deviation about the median of the following observation:
22, 24, 30, 27, 29, 31, 25, 28, 41, 42

Ans. : Formula used for mean deviation:

$$\mathrm{MD} = \frac{1}{n}\sum_{i=1}^{n}|d_i|$$

Here, n = 10

Also, Median is the AM of the fifth and the sixth observation.

Median, $M = \frac{28+29}{2} = 28.5$

X_i	$\lvert d_i \rvert = \lvert x_i - M \rvert$
22	6.5
24	4.5
30	1.5
27	1.5
29	0.5
31	2.5
25	3.5
28	0.5
41	12.5
41	13.5
Total	**47**

$\mathrm{MD} = \frac{1}{10} \times 47 = 4.7$

110. For a group of 200 candidates, the mean and standard deviations of scores were found to be 40 and 15 respectively. Later on it was discovered that the scores of 43 and 35 were misread as 34 and 53 respectively. Find the correct mean and standard deviation.

Ans. : We have,

$n = 200, \overline{X} = 40, \sigma = 15.$

$\therefore \overline{X} = \frac{1}{n}\sum x_i = \overline{X} = 200 \times 40 = 8000.$

Corrected $\sum x_i$ = Incorrected $\sum x_i$ - (sum of incorrect values) + (sum of correct values)

= 8000 - 34 - 53 + 43 + 35 = 7991

$\therefore \text{Corrected mean} = \frac{\text{corrected} \sum x_i}{n} = \frac{7991}{200} = 39.955$

Now $\sigma = 15$

$\Rightarrow 15^2 = \frac{1}{200}\left(\sum {x_i}^2\right) - \left(\frac{1}{200}\sum x_i\right)^2$

$\Rightarrow 255 = \frac{1}{200}\left(\sum {x_i}^2\right) - \left(\frac{8000}{200}\right)^2$

$\Rightarrow 255 = \frac{1}{200}\left(\sum {x_i}^2\right) - 1600$

$\sum {x_i}^2 = 200 \times 1825 = 365000$

Incorrect $\sum {x_i}^2 = 365000$

corrected $\sum x_i{}^2$ = $\left(\text{incorrected} \sum x_i{}^2\right)$ - (sum of squers of incorrect values) + (sum of squers of correct values)

= 365000 - $(34)^2$ - 53^2 + $(43)^2$ + 35^2 = 364109

so, Corrected $\sigma = \sqrt{\frac{1}{n}\sum x_i{}^2 - \left(\frac{1}{n}\sum x_i\right)^2} = \sqrt{\frac{364109}{200} - \left(\frac{7991}{200}\right)^2}$

$= \sqrt{1820.545 - 1596.402} = 14.97$

111. Determine the mean and standard deviation for the following distribution:

Marks	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Frequency	1	6	6	8	8	2	2	3	0	2	1	0	0	0	1

Ans. :

Marks	f_i	$f_i x_i$	$d_i = x_i - \bar{x}$	$f_i d_i$	$f_i d_i^2$
2	1	2	-4	-4	16
3	6	18	-3	-18	54
4	6	24	-2	-12	24
5	8	40	-1	-8	8
6	8	40	-1	-8	8
7	2	14	1	2	2
8	2	16	2	4	8
9	3	27	3	9	27
10	0	0	4	0	0
11	2	22	5	10	50
12	1	12	6	6	36
13	0	0	7	0	0
14	0	0	8	0	0
15	0	0	9	0	0
16	1	16	10	10	100
Total	$\sum f_i = 40$	$\sum f_i x_i = 239$		$\sum f_i d_i = -1$	$\sum f_i x_i^2 = 325$

$\therefore$ Mean $\bar{x} = \frac{\sum f_i x_i}{\sum f_i} = \frac{239}{40} = 5.975 \approx 6$

and $\sigma = \sqrt{\frac{\sum f_i x_i}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i}\right)^2}$

$= \sqrt{\frac{325}{40} - \left(\frac{-1}{40}\right)^2}$

$= \sqrt{8.125 - 0.000625}$

$= \sqrt{8.124375}$

$= 2.85$

112. The mean and standard deviation of some data for the time taken to complete a test are calculated with the following results: Number of observations = 25,

mean = 18.2 seconds, standard deviation = 3.25 seconds. Further, another set of 15 observations x_1, x_2, ..., x_{15}, also in seconds, is now available and we have $\sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$. Calculate the standard derivation based on all 40 observations.

Ans. : Given $n_1 = 25,\ \bar{x}_i = 18.2,\ \sigma_1 = 3.25$

$n_2 = 15,\ \sum_{i=1}^{15} x_i = 279$ and $\sum_{i=1}^{15} x_i^2 = 5524$

For first set $\sum x_i = 25 \times 18.2 = 455$

$\therefore\ \sigma_1^2 = \frac{\sum x_i^2}{25} - (18.2)^2$

$\Rightarrow (3.25)^2 = \frac{\sum x_i^2}{25} - (18.2)^3 2$

$\Rightarrow 10.5625 + 331.24 = \frac{\sum x_i^2}{25}$

$\Rightarrow \sum x_i^2 = 25 \times (10.5625 + 331.24)$

$\Rightarrow \sum x_i^2 = 25 \times 341.8025 = 8545.0625$

For combined SD of the 40 observations, n = 40

Now, $\sum_{i=1}^{40} x_i^2 = 5524 + 8545.0625 = 14069.0625$

and $\sum_{i=1}^{40} x_i = 455 + 279 = 734$

$\therefore\ \mathrm{SD} = \sqrt{\frac{14069.0625}{40} - \left(\frac{734}{40}\right)^2}$

$= \sqrt{351.1726 - (18.35)^2}$

$= \sqrt{351.726 - 336.7225}$

$= \sqrt{15.0035}$

$= 3.87$

113. While calculating the mean and variance of 10 readings, a student wrongly used the reading 52 for the correct reading 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and the variance.

Ans. : Given, $n = 10,\ \bar{x} = 45,\ \sigma^2 = 16$

$\bar{x} = 45$

$\Rightarrow \frac{\sum x_i}{n} = 45$

$\Rightarrow \sum x_i = 450$

Corrected $\sum x_i = 450 - 52 + 25 = 423$

$\therefore$ Corrected mean, $\bar{x} = \frac{432}{10} = 42.3$

$\Rightarrow\ \sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n}\right)^2$

$\Rightarrow 16 = \frac{\sum x_i^2}{10} - (45)^2$

$\Rightarrow \sum x_i^2 = 20410$

$\therefore$ Corrected $\sum x_i^2 = 20410 - (53)^2 + (25)^2 = 18331$

And Corrected $\sigma^2 = \frac{18331}{10} - (42.3)^2 = 43.81$

114. The mean life of a sample of 60 bulbs was 650 hours and the standard deviation was 8 hours. A second sample of 80 bulbs has a mean life of 660 hours and standard deviation 7 hours. Find the overall standard deviation.

Ans. : Given that $n_1 = 60,\ \bar{x}_1 = 650,\ s_1 = 8$

And $n_2 = 80,\ \bar{x}_2 = 660,\ s = 7$

We know that for a combined series.

$$\sigma = \sqrt{\frac{n_1 s_1^2 + n_2 s_2^2}{n_1 + n_2} + \frac{n_1 n_2 (\bar{x}_1 - \bar{x}_2)^2}{(n_1 + n_2)^2}}$$

$$= \sqrt{\frac{60 \times (8)^2 + 80 \times (7)^2}{60 + 80} + \frac{60 \times 80 (650 - 660)^2}{(60 + 80)^2}}$$

$$= \sqrt{\frac{6 \times 64 + 8 + 49}{14} + \frac{60 \times 80 \times 100}{140 \times 140}}$$

$$= \sqrt{\frac{192 + 196}{7} + \frac{1200}{49}} = \sqrt{\frac{338}{7} + \frac{1200}{49}}$$

$$= \sqrt{\frac{2716 + 1200}{49}} = \sqrt{\frac{3916}{49}}$$

$$= \frac{62.58}{7} = 8.9$$

Hence, the required SD = 8.9

115. Calculate the mean deviation about the mean of the set of first n natural numbers when n is an odd number.

Ans. : First n natural numbers are 1, 2, 3, ..., n. Here, n is odd.

$$\therefore \text{ Mean } \bar{x} = \frac{1 + 2 + 3 + \ldots\ldots + n}{n} = \frac{\frac{n(n+1)}{2}}{2} = \frac{n+1}{2}$$

The deviations of numbers from mean $\left(\frac{n+1}{2}\right)$ are

$$1 - \frac{n+1}{2}, 2 - \frac{n+1}{2}, 3 - \frac{n+1}{2}, \ldots\ldots\ldots, n - \frac{n+1}{2}$$

$$\text{i.e.,} -\frac{n-1}{2}, \frac{n-3}{2}, \ldots\ldots, -2, -1, 0, 1, 2, \ldots\ldots, \frac{n-1}{2}$$

The absolute values of deviation from the mean i.e., $|x_i - \bar{x}|$

$$\frac{n-1}{2}, \frac{n-3}{2}, \ldots\ldots, 2, 1, 0, 1, 2, \ldots\ldots, \frac{n-1}{2}$$

The sum of absolute values of deviations from the mean i.e. $|x_i - \bar{x}|$

$$= 2\left(1 + 2 + 3 + \ldots\ldots \text{ to } \frac{n-1}{2} \text{terms}\right)$$

$$= 2 \cdot \frac{\frac{n-1}{2}\left(\frac{n-1}{2} + 1\right)}{2} = \frac{n-1}{2} \cdot \frac{n+1}{2} = \frac{n^2 - 1}{4}$$

$\therefore$ Mean deviation about the mean.

$$= \frac{\sum |x_i - \bar{x}|}{n} = \frac{\frac{n^2-1}{4}}{n} = \frac{n^2 - 1}{4n}$$

116. Mean and standard deviation of 100 observations were found to be 40 and 10, respectively. If at the time of calculation two observations were wrongly taken

as 30 and 70 in place of 3 and 27 respectively, find the correct standard deviation.

Ans. : Given that, $\mathrm{n}=100,\ \bar{\mathrm{x}}=40,\ \sigma=10$

$\therefore\ \bar{\mathrm{x}}=\frac{\sum \mathrm{x_i}}{\mathrm{N}}$

$\Rightarrow 40=\frac{\sum \mathrm{x_i}}{100}$

$\Rightarrow \sum \mathrm{x_i}=4000$

Corrected $\sum \mathrm{x_i}=4000-30-70+3+27=3930$

and Corrected mean $=\frac{3930}{100}=39.3$

Now, $\sigma^2=\frac{\sum \mathrm{x_i^2}}{\mathrm{n}}-(40)^2$

$\Rightarrow 100=\frac{\sum \mathrm{x_i^2}}{100}-1600$

$\Rightarrow \sum \mathrm{x_i^2}=1700\times 100$

$\Rightarrow \sum \mathrm{x_i^2}=170000$

$\therefore\ \text{Corrected} \sum \mathrm{x_i^2}=170000-(30)^2-(70)^2+(3)^2+(27)^2$

$=170000-900-4900+9+729=164938$

$\therefore\ \text{Correct SD}=\sqrt{\frac{164938}{100}-(39.3)^2}$

$=\sqrt{1649.38-1544.49}$

$=\sqrt{104.89}=10.24$

----- CSC- Choose the goal ,stick on it and complete it -----